Making Math Matter: Project-based Learning in Mathematics

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Abstract
We examine key components of project-based learning (PBL) and explore how 21st Century skills such as critical thinking, communication, and collaboration are embedded in a sample Algebra 2 PBL unit. PBL-related resources are provided to inspire readers to design PBL units of their own.

Keywords
Project-based Learning, Inquiry, 21st Century Skills.

INTRODUCTION

Over the past two decades, various reform documents (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association and Council of Chief State School Officers, 2010) have emphasized the importance of students' understanding of mathematics content and also the ways in which students engage in the learning of mathematics. These documents advocate for student engagement in authentic problem solving. Dan Meyer (2010) states traditional mathematics textbooks often present math problems that do not engage students in reasoning and problem solving. Instead, problems are usually presented such that a compelling problem is broken down for the student, paving a smooth straight path from one step to another. In this paper, we focus on a driving question: How can teachers support students in deeper, more meaningful learning of mathematics?

One curricular and instructional model that focuses on increasing the range of students' interests as well as their conceptual understanding of mathematics content is project-based learning (PBL). PBL is an inquiry-based instructional approach that reflects a learner-centered environment and concentrates on students' application of disciplinary concepts, tools, experiences, and technologies to answer questions and solve real-world problems (Krajcik & Blumenfeld, 2006; Markham, Larmer, & Ravitz, 2003).

PROJECT-BASED LEARNING

We embrace the Buck Institute for Education's definition of PBL: *In project-based learning, students go through an extended process of inquiry in response to a complex question, problem or challenge. While allowing for some degree of student "voice and choice," rigorous projects are carefully planned, managed, and assessed to help students learn key academic content, practice 21st Century Skills (such as collaboration, communication, and critical thinking), and create high-quality, authentic products and presentations* (BIE, 2013).

Some of the general core principles and practices of PBL are:

1. There is a professional culture of trust, respect, and responsibility among the learners themselves and the teacher in a PBL environment.
2. PBL units focus on 21st Century Skills as well as academic standards such as the Common Core State Standards for Mathematics.
3. Scaffolding activities in PBL units include student-centered instruction to increase relevance and rigor.
4. PBL units are designed to connect learning to other content subject areas and to the post-high school world.
5. PBL units infuse technology as a tool for communicating, collaborating, and learning.
6. PBL units draw in partnerships with community institutions such as higher education, businesses, and non-profit agencies.

Doing Projects Compared To PBL

The traditional notion of doing projects usually places them at the end of a unit, after the teacher has taught a series of lessons and students have been pushed through homework assignments, lectures, and readings. Students then demonstrate their understanding of the content by completing a culminating project (see Figure 1). Practice problems, lecture, textbook activities, and class discussions are examples of what might happen in a traditional math classroom when the content is being presented.

In a PBL classroom, the project is not at the end of the unit. Instead, an entry event launches the project at the beginning of a unit and students are pulled through the curriculum by a driving question and authentic problem that creates a need to know the content of the unit (see Figure 2).

Figure 1. Timeline of Doing Projects

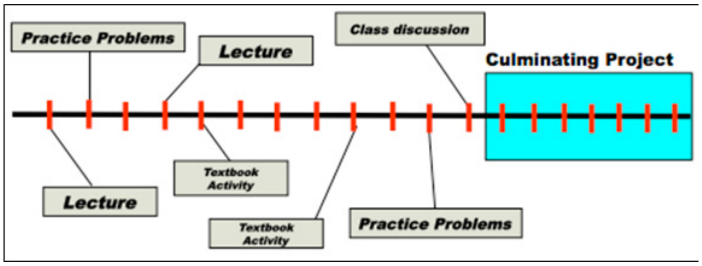
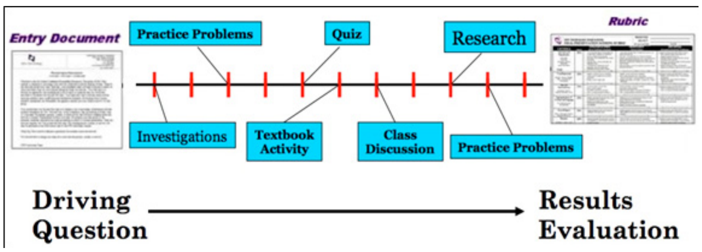


Figure 2. Timeline of PBL



The same elements found in a traditional classroom such as practice problems, textbook activities, and class discussions are integrated into the unit in response to students' "need to knows" based on the entry event. Lectures might be replaced with students investigating, building, and researching concepts so inquiry is taking place. Articulating how this works is perhaps best illustrated by an example PBL unit titled "Interest in Interest," an Algebra 2 PBL unit created by Crystal Collier, an Indiana teacher. The timeline for this project is presented in Table 1. We also include the project planning form for this unit as an appendix.

What follows is a presentation of some of the essential elements of a PBL unit, illustrated by excerpts from "Interest in Interest".

Entry Event

A PBL unit is most often launched with an entry event that helps contextualize the problem and motivates students to engage in the content. Figure 3 illustrates an entry event that is in the form of a document—a letter written from a company asking for students' help. Ideally, the problem is an authentic one and a representative of the company will present the challenge either virtually or in person.

Table 1. Project Calendar for "Interest in Interest" Algebra 2 PBL Unit

Day 1	Introduction of the project: Present the problem/ challenge to the students. Review expectations of the project.
Day 2	Car buying criteria workshop: What criteria do we need to consider before picking a car?
Day 3	Car buying resources technology workshop: Show various websites on car information. Students revisit car choices based on Day 2 and 3 workshops. Students compile and augement their lists using Word or Excel.
Day 4	Affordability workshop: How much can I spend monthly on car payments based on my salary? Students refine car choices based on Day 2, 3, and 4 workshops. Students research auto loan rates and calculate monthly payment of cars.
Day 5	Quiz Review Compound interest workshop: Exploration of the impact of compund interest for savings, CD, and loan rates.
Day 6	Depreciation workshop: Compare Kelley Blue Book depreciation car values vs. continuous compounding values with 15% depreciation rate. Technology workshop: Use computer spreadsheet to plot points on graph to generate a possible function that models the data.
Day 7	Technology workshop: Use Excel and PowerPoint effectively. Develop PowerPoint slides for presentation. Review expectations of presentation.
Day 8	Group computer lab work time. Students practice presentations. Teacher checks in with groups' progress.
Day 9	Final presentations.
Day 10	Students assess the productivity of themselves and peers. Class relfects and debriefs over the unit: Discuss recommendations for performance and project improvement.

An entry event should accomplish at least four things that are critical to a successful project:

1. Hook the students;
2. Allow students to discern their role;
3. Lay out the project or problem to be completed or solved; and
4. Provide information that will motivate the students to ask questions and seek answers in honor of the standards and skills useful for formulating a response or solution to the problem.

Entry events may use documents, video, presentations, or any other activity to engage students.

Figure 3. Entry Event of "Interest in Interest" Algebra 2 PBL Unit

Dear Student:

As you know, modern consumers are subjected to a seemingly endless stream of advertisement and purchase pressures. So, it's more important than ever to learn how to live within your means, because your purchase choices now can greatly affect your financial future.

Buying a first car is a very exciting prospect to young people as they enter adulthood. Cars play a number of different roles in our modern American culture. A car is a tool for both getting things done and for having fun, and a symbol of self-expression. It's also probably one of the biggest purchases you've ever made, and the consequences of that purchase, good or bad, can affect you for years to come.

P. Beale Savings & Loan has a proud tradition of helping get responsible young adults like you behind the wheel with our competitive auto loans and supportive financial counseling. We are committed to helping you make a financial decision you can be proud of, but to do that effectively we first need YOUR help. We need you to step into the role of a new college graduate looking to buy his or her first car, and to answer the following question: How can we, as recent college graduates, determine the best vehicle purchase for our income?

By using a methodical approach and doing research, and based on your own expected income level as a college graduate, come up with a budget, find an lender (it doesn't have to be us) with interest rates and monthly payments you can afford, and choose the best vehicle that fits that budget. By "best vehicle" we mean a car that is affordable, holds its value, and will serve both your needs and personal desires.

We then ask that you prepare a simple presentation showing both the vehicle you chose, and the criteria that led to that decision. Using this data, we hope to help guide future college grads to making the best car-buying decision they can make. We also sincerely hope that by taking part in this project, you'll empower yourself to make your own great car purchase!

We look forward to your presentations and wish you good luck as you begin working. You have the opportunity to change lives through this project and we hope that you do.

Best Wishes,

Preston Beale Jr.
Assistant Vice President
P. Beale Savings & Loan

Driving Question

After engaging students in the entry event, they should be able to articulate the problem statement or the Driving Question of the project. The Driving Question is an open-ended challenge or problem that focuses learners' work and deepens their learning by centering on questions and/or problems, significant issues, and debates. Teachers can guide students to define the problem statement within the Driving Question by having them reflect on the following framework: *How do we as… (student's role) create/ research/develop… (task) so that… (desired outcome).*

The Driving Question for "Interest in Interest", our example unit, is: How does income impact a consumer's ability to make large purchases? This driving question requires the project designer to articulate a scenario that can be meaningful to students. Thus, the problem statement is: How can we, as recent college graduates, determine the best vehicle purchase for our income? The students conducting research for this PBL Unit are in an Algebra 2 class and come from an early college career setting. By framing their position as recent college graduates seeking a vehicle purchase, there is a dual layer of authenticity and adult connections made for the students.

After students articulate the Driving Question with the teacher's guidance, students practice an essential problem solving skill by determining and recording what they will need to know in order to answer the Driving Question. This list becomes a living document for the duration of the project. Students add to this list and it is revisited daily through various formats to assess the progress of the class and to allow student voice to determine the next steps for investigation which highlight the relevance of upcoming instruction.

Scaffolding Instruction

Ways in which the teacher supports students learning (i.e., practice problems, textbook activities, class discussions, investigations, research, etc.) and the problem solving process are referred to as scaffolding techniques. Scaffolds are integrated into the instruction of the unit as the students need the information so their learning becomes authentic and relevant. Equally important, students' learning of the context of the project should be well balanced with the content. For example, in the "Interest in Interest" Algebra 2 Unit, students need to choose the most affordable vehicle (context), and also master the concepts of exponential and logarithmic functions (content). Also important is that PBL encourages learning that is inquiry-based and in which the inquiry should lead learners to construct something new—an idea, an interpretation, a new way of displaying what they have learned. We

Table 2. Anticipated Need-to-Knows

Anticipated **Need to Know**	**Scaffolding activities** to address NTK	**Assessment** for assingmment/ activity/action	**Learning Outcomes** addressed in assignment
How will this be graded?	Rubric will be given to students and explained.	Students will be asked to rewrite the rubric. Students will demonstrate understanding of the rubric by being able to transfer it into "kit-friendly" language	I can explain, in my own words, the medithods by which I will be assessed in this project.
What is a case study?	Students will be given a sample case study to review. We will discuss the structure and the methods used to generate and analyze relevant data.	Students will create an original case study utilizing methods similar to those used in the example.	I can generate accurate data and I can properly interpret my findings as part of a study.
What types of graphs do I need to include?	Students will have a workshop devoted to graphing and developing meaningful graphs.	Students will be asked to use their case study to create at least 3 different graphs illustrating different savings strategies.	I can draw a graph to illustrate how the banking product works over time given certain variables.
What is a professional presentation?	Students will have a workshop dedicated to developing a concept for their presentations and will receive feedback to help them develop that concept to a professional level.	Students will have to submit a rough draft or outline of their presentation with a prototype copy of the original marketing material to be presented. Students will be given the opportunity to revamp if needed to implement feedback. Students will demonstrate a mastery understanding of professional presentation through the execution of their own presentations and materials.	I can present my ideas in a clear, concise and professional manner.
How do I use Excel?	Students will have a workshop designed to teach them how to make use of excel to generate data and to create graphs for their project.	Students will have an assignment focused on graphing and using excel to create graphs. Their graphs must be accurate and relevant to the case study.	I can use excel to create a variety of graphs illustrating various logarithmic and exponential functions.
How do I use PowerPoint?	Students will have a workshop where they learn to embed objects in PowerPoint to create a presentation of their Case Study.	Students will use their PowerPoint as the focal point of their presentations of their case study.	I can create a PowerPoint slide show that engages the audience in my case Study.
What math will I need to know?	Students will have two to three workshops on exponential functions and logarithmic functions. These topics will be addressed in context to the project.	Students will take quizzes and a test to determine the level of learning they have achieved on these learning goals. Students will also demonstrate their learning through the final presentation by presenting accurate data which they are able to properly interpret. Students will also be able to answer questions from the panel regarding their findings and support their answers with mathematical data.	I can accurately solve and interpret logarithmic and exponential equations.

provide an example of how Crystal Collier anticipated some of the "need to knows" her students would have, and how she planned to support students' learning. These "need to knows" are not exhaustive, but merely examples to indicate the level of detail required in the PBL unit planning process.

21st Century Skills

Learners need to do much more than remember information in a PBL environment—they need to use higher-order thinking skills. They learn to work as a team and contribute to a group effort. They must listen to others and make their own ideas clear when speaking, be able to read a variety of material, write, or otherwise express themselves in various modes, and make effective presentations. These skills, competencies, and habits of mind are often known as 21st Century Skills. Various 21st Century Skills include: communication, creativity, use of technology, group process and collaboration, problem solving and critical thinking, and task- and self- management (Markham et al., 2003, pp. 25-27). In the Algebra 2 Unit, collaboration, presentation, digital age literacy, and critical thinking and reasoning are explicitly taught and assessed; time management and investing thinking skills are encouraged by project work, but not taught or assessed. Learners work independently and take responsibility when they are asked to make choices. The opportunity to make choices, and to express their learning in their own voice, also helps increase learners' educational engagement.

Rubric

Rubrics help students understand the expectations of the project and prepare them for how they will demonstrate their learning for public scrutiny and critique. The rubric is designed so that the PBL unit not only has students demonstrate content mastery, but soft skills (i.e., 21st Century Skills) as well. Even though various soft skills may be encouraged, Larmer, Ross, and Mergendollar (2009) recommend novice PBL practitioners to identify no more than two soft skills if those skills are explicitly being taught throughout the unit and assessed as outcomes in the projects. The example rubric in Figure 4 illustrates the various criteria students must meet: content mastery, the 21st Century Skill of critical thinking and reasoning, and presentation skills.

Figure 4. "Interest in Interest Algebra 2 PBL Rubric

Criteria	Emerging	Developing	Proficient	Advanced
Communication & Representation (8 Points) *What is the evidence that the student can communicate mathematical ideas to others?*	• Uses representations (diagrams, tables, graphs, formulas) in ways that **confuse the audience** • Provides **inaccurate mathematical explanation** of their graphs	• Uses representations (diagrams, tables, graphs, formulas), though correct, **do not help the audience** follow the chain of reasoning, extraneous representations may be included • Uses **imprecise mathematical explanations** with missing units of measure or labeled axes	• Uses multiple representations (diagrams, tables, graphs, formulas) to **help the audience** follow the chain of reasoning • With few exceptions, provides **accurate mathematical explanation** including **units of measure and labeled axes**	• Uses multiple representations (diagrams, tables, graphs, formula) **and key explanations to enhance the audience's understanding** of the solution; only relevant representations are included. • Uses **precise definitions and accurate representations** including units of measure and labeled axes; uses **formal notation**
	1--2	3--4	5--6	7--8
Reasoning & Proof (8 Points) *What is the evidence that the student can apply mathematical reasoning/procedures in an accurate and complete manner?*	• Provides **incorrect solutions without** justifications • Results are not interpreted in terms of context	• Provides **partially correct solutions or correct solution** without logic or justification • Results are interpreted partially or incorrectly in terms of context	• Constructs **logical, correct, complete solution** • Results are interpreted correctly in terms of context	• Constructs **logical, correct, complete solution with justifications** • Interprets results correctly in terms of context, indicating the domain to which the solution applies
	1--2	3--4	5--6	7--8
Connections (8 Points) *What is the evidence that the student understands the relationships between the concepts, procedures, and/or real-world applications inherent in the problem?*	• **Does not provide** a model • **Little or no evidence** of applying previous math knowledge to given problem	• **Creates a limited model** to simplify a complicated situation • **Applies previous math knowledge** to given problem but may include **reasoning or procedural errors**	• **Creates a model** to simplify a complicated situation • **Applies and extends math previous knowledge** correctly to given problem	• **Creates a model** to simplify a complicated situation and identifies limitations of model • **Applies and extends previous knowledge correctly** to given problem; **makes appropriate use of derived results**
	1--2	3--4	5--6	7--8

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***This rubric is a modified and adapted version of New Tech Network's Knowledge and Thinking Rubric for Math Problem Solving, Grade 10**

RESOURCES

We list a variety of resources in Table 3 that we have found useful as we design PBL units for mathematics. Some resources provide sample PBL units; these units may be brief ideas or very detailed units. Other resources provide sample videos to support practitioners to implement PBL units. Research studies are also showcased in some resources to help investigate the effectiveness of PBL, while other resources provide implementation tips and/or strategies for PBL practitioners. Lastly, the column marked "ideas that drive design" are resources that contain problems/challenges that may inspire a PBL unit topic.

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Table 3. Various PBL Resources

	Sample PBL Units	Sample Videos	Research	Tips for using PBL	Ideas that drive design
Buck Institute for Education http://www.bie.org	X	X	X		
Project-based Learning Handbook (Markham et al., 2003)	X			X	X
PBL Start Kit (Larmer et al., 2009)	X			X	X
Edutopia http://www.edutopia.org/project-based-learning				X	X
Indiana Collaborative for Project-based Learning http://www.rose-prism.org/moodle/	X				
The PBL Academy http://iuemoodle.educ.indiana.edu/moodle/	X				
Curriki PBL Geometry http://www.curriki.org/welcome/resources-curricula/curriki-geometry-course/	X				
Innocentive http://www.innocentive.com					X
Mathalicious http://www.mathalicious.com					X
Emergent Math http://emergentmath.com/my-problem-based-curriculum-maps/					X

throughout her career has been the professional development of teachers of mathematics. Since 2009, she has been leading the PBL Academy, (originally Math Matters), an initiative in the southeastern counties of Indiana to introduce project-based learning (PBL) techniques to educators at all grade levels.

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Acknowledgements:
Thanks to Crystal Collier for letting us showcase her Algebra 2 PBL unit. Any inquiries on the unit should be sent directly Crystal Collier (CrystalLaurey@gmail.com).

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